Spin-wave mediated quantum corrections to the conductivity in thin ferromagnetic gadolinium films

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We present a study of quantum corrections to the conductivity of thin ferromagnetic gadolinium films. *In situ* magneto-transport measurements were performed on a series of thin films with thickness $d < 135A$. For sheet resistances $R_0 < 4011\Omega$ and temperatures $T < 30K$, we observe a *linear* temperature dependence of the conductivity in addition to the logarithmic temperature dependence expected from well-known quantum corrections in two dimensions. We show that such a linear T-dependence can arise from a spin-wave mediated Altshuler-Aronov type correction.

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Recent studies of the anomalous Hall effect in thin Fe films [1] have provided strong evidence for weak localization (WL) effects in disordered ferromagnetic films. Since WL effects are cut off by various temperature independent phase breaking scatterings and especially by the magnetic field inside the ferromagnet, one needs a sufficiently large temperature dependent phase relaxation rate $1/\tau_\phi$ to have an experimentally accessible disorder and temperature interval where such effects can be observed. While the contribution to $1/\tau_\phi$ in ferromagnetic films from electron-electron interactions is small, a much larger contribution is obtained from scattering off spin-waves [2, 3], such that the characteristic logarithmic temperature dependence of the conductivity due to WL effect was observed in polycrystalline Fe films within a range of temperature $5K < T < 20K$, for sheet resistances $R_0 < 3k\Omega$.

Given the importance of spin-waves in Fe films, one expects to see an even larger effect in other ferromagnetic films with larger magnetic moments. In particular, quantum corrections to the conductivity due to scattering off spin-waves should be observable, just like the quantum corrections due to electron-electron Coulomb interactions, if the exchange coupling is large enough and the spin-wave gap is smaller than the temperature. It turns out that an excellent candidate is gadolinium (Gd), with a spin-wave gap of about 30 mK and a Curie temperature of $293K$ [4, 5].

We have carried out systematic *in situ* magnetotransport measurements on a series of Gd films with varying thicknesses ($35\mathring{A} < d < 135\mathring{A}$) having sheet resistances R_0 ranging from 428Ω to 4011Ω. For temperatures $5K < T < 30K$, we observe simultaneous presence of two types of quantum corrections to the Drude conductivity: one has the expected logarithmic temperature dependence that is a hallmark of quantum corrections in two-dimensions [6]; the other has a heretofore-unobserved approximately linear temperature dependence, which we attribute to a spin-wave mediated Altshuler-Aronov type correction to the conductivity. We calculate this spin-wave contribution within a standard diagrammatic perturbation theory and show that the results agree with the experimentally observed temperature dependence.

A series of ultrathin films of Gd in the Hall bar geometry was grown by r.f. magnetron sputtering through a shadow mask onto sapphire substrates held at a temperature of 250 K. The current and voltage leads of the deposited sample overlapped with predeposited palladium contacts, thus allowing reliable electrical connection for *in situ* measurements of the electrical properties. The experiments were performed in a specialized apparatus in which the sample can be transferred without exposure to air from the high vacuum deposition chamber to a 7 T magnet located in a low temperature cryostat. To parameterize the amount of disorder in a given film[1], we use sheet resistances $R_0 \equiv R_{xx}(T = 5K)$ where R_{xx} is the longitudinal resistance. R_0 spans the range from 428Ω (135 Å thick) to 4011Ω (35 Å thick). Longitudinal and Hall resistances were measured using standard four-probe lock-in techniques for low resistance samples ($R_0 \leq 2613\Omega$) and dc techniques for higher resistance samples($R_0 > 2613\Omega$).

The inset of Fig. 1 shows the temperature dependence of the longitudinal conductivity σ_{xx} for a series of thin Gd films with varying sheet resistances R_0 listed in the legend. A fitting function of the form

$$\sigma_{xx}/L_{00} = P_1 + P_2 \ln(T/T_0) + P_3 \,(T/T_0)^p \,, \qquad (1)$$

where $L_{00} = e^2/\pi h$ and $T_0 = 5K$ is a reference temperature, has been used to fit each of the curves. An expanded view of a typical fit (solid red line) is shown in the main panel for the $R_0 = 428\Omega$ data (open squares). The fitting parameters for all of the curves are shown in Table I with respective errors in parentheses.

There are several important points about the fits that are worth emphasizing. First, note that the tempera-

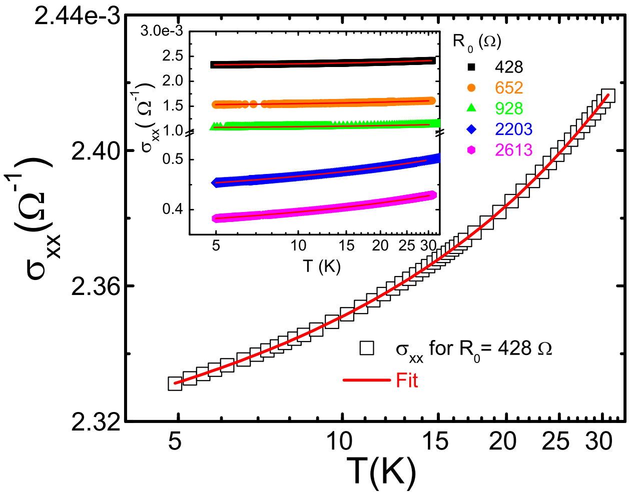

FIG. 1: Temperature dependence of σ_{xx} for a series of Gd thin films (inset) with various sheet resistances R_0 listed in the legend. Note that the temperature scales are logarithmic. The fits for each curve are obtained using Eq. (1), with fitting parameters listed in Table I. The main panel in an expanded view shows the quality of fit for the $R_0 = 428\Omega$ data.

ture scale is logarithmic, and it is clear that a $\ln T$ alone can not fit the data. Second, The power p is close to 1 for $R_0 < 3k\Omega$, but then changes significantly when $R_0 > 4k\Omega$. Third, the coefficient P_3 is roughly independent of disorder. We will show that the unusual temperature dependence is consistent with a sum of contributions from well-known quantum corrections in two-dimensions and a novel spin-wave mediated correction analogous to the Altshuler-Aronov electron-electron contribution in disordered systems [7]. While the Altshuler-Aronov contribution gives rise to a logarithmic temperature dependence in two dimensions, we show that the spin-wave mediated contribution can be linear in temperature within certain ranges of the parameters, consistent with the experiments. The theory ceases to be valid for large disorder ($R_0 > 4k\Omega$), where the temperature dependence is no longer linear.

To make comparisons with earlier studies on Fe films [1, 8], we have also measured the Anomalous Hall (AH) resistances of the Gd films at 7 Tesla magnetic field. Following Ref. [8], we define normalized relative changes

$$\Delta^N(Q_{ij}) \equiv (1/L_{00}R_0)(\delta Q_{ij}/Q_{ij}) \tag{2}$$

with respect to a reference temperature $T_0 = 5K < T$, where $\delta Q_{ij} = Q_{ij}(T) - Q_{ij}(T_0)$ and Q_{ij} refers to either resistances R_{xx}, R_{xy} or conductivity σ_{xx}, σ_{xy}.

Figure 2 shows $\Delta^N(\sigma_{xy})$, together with $\Delta^N(R_{xx})$ and $\Delta^N(R_{xy})$ for comparison, for two different sheet resistances. We find that within the range of disorder, $\Delta^N\sigma_{xy} \approx 0$ for $5K < T < 20K$. As shown in Ref. [9], the interaction correction to the AH conductivity is exactly zero, due to symmetry reasons. This is true for both repulsive Coulomb interaction and the attractive spin-wave mediated interaction. However, the WL correction to the AH conductivity need not be zero. In fact, the total WL contribution is given by

$$\Delta^N\sigma_{xy}^{WL} = \frac{1}{1+r_{xy}} \ln\frac{T}{T_0}; \quad r_{xy} \equiv \frac{\sigma_{xy}^{sj}}{\sigma_{xy}^{ss}} \tag{3}$$

where r_{xy} is the ratio of two different mechanisms contributing to the AH conductivity, namely the side-jump [11] σ_{xy}^{sj} and the skew scattering [10] σ_{xy}^{ss}. Thus the ratio r_{xy} is a non-universal quantity. It turns out that for Fe films, while $r_{xy} >> 1$ for films deposited on glass, the opposite $r_{xy} << 1$ is true for films deposited on sapphire [1] or on antimony [8]. Our current results $\Delta^N\sigma_{xy} \approx 0$ for Gd deposited on sapphire agree with those of Fe films deposited on the same substrate. As for the longitudinal part, the coefficient A_R defined by

$$\Delta^N\sigma_{xx} = A_R \ln(T/T_0) \tag{4}$$

is given as $A_R = A_R^{WL} + A_R^I = 1 + h_{xx}$ where the first term ($A_R^{WL} = 1$) is due to WL and the second term

$R_0(\Omega)$	P_1	P_2	P_3	p
428	187.88(1)	0.79(1)	0.97(1)	1.039(6)
652	123.24(1)	0.67(1)	1.03(1)	0.976(5)
928	86.47(1)	0.71(1)	0.72(2)	0.934(8)
2203	36.10(2)	0.75(1)	0.65(2)	0.84(1)
2613	30.25(1)	0.70(1)	0.72(1)	0.818(4)

TABLE I: Fitting parameters defined in Eq. (1) used in Fig. 1

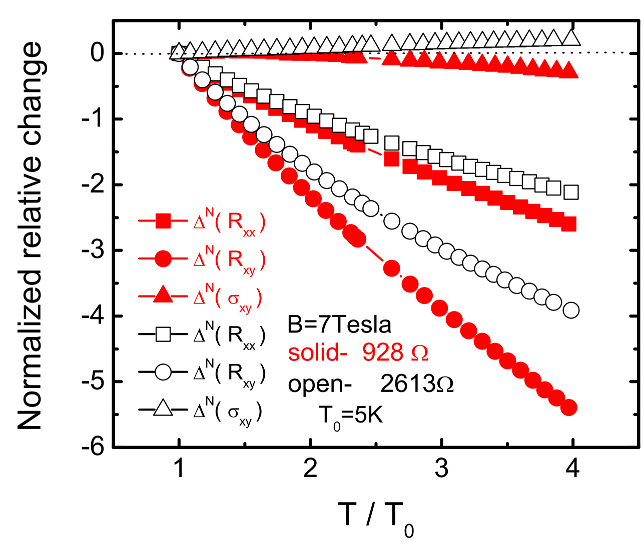

FIG. 2: Normalized relative changes $\Delta^N(\sigma_{xy})$, defined in Eq. (2), for two different sheet resistances. For comparison we also show $\Delta^N(R_{xx})$ and $\Delta^N(R_{xy})$.

$(A_R^I = h_{xx})$ is the exchange plus Hartree interaction contribution, with $h_{xx} = (1 - 3\tilde{F})/4$ where $\tilde{F}$ is the Hartree term. It has been argued that at least for Fe films, the total interaction correction h_{xx} is very small due to a near cancellation of the exchange and Hartree terms, which is expected due to strong screening. This results in $A_R \approx 1$ for Fe films. Table I shows that $A_R \approx 0.75$ for our Gd films. This suggests that h_{xx} may actually be negative, due to an even larger Hartree contribution. Since both r_{xy} and h_{xx} are non-universal quantities, we only note that a large r_{xy} (similar to Fe films on saphhire) and a small negative h_{xx} (large Hartree term, again similar to Fe films) are consistent with the current experimental observations (on Gd films on sapphire).

To understand the linear T-dependence of the longitudinal conductivity, we evaluate the spin wave contributions within the standard diagrammatic perturbation theory. The film is described as a quasi-two-dimensional system of conduction electrons with Fermi energies $\epsilon_{F\sigma}$ depending on the spin index $\sigma =\uparrow, \downarrow$. We model the total impurity potential as a sum over identical single impurity potentials $V(\mathbf{r} - \mathbf{R}_j)$ at random positions $\mathbf{R}_j$. The Hamiltonian is given by

$$\begin{aligned} H &= \sum_{\mathbf{k}\sigma} (\epsilon_{\mathbf{k}} - \frac{1}{2}\sigma B) c^{+}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} \\ &+ \sum_{\mathbf{k},\mathbf{k\prime}\sigma,\mathbf{j}} V(\mathbf{k} - \mathbf{k}') e^{i(\mathbf{k}-\mathbf{k}')\cdot \mathbf{R}_j} c^{+}_{\mathbf{k\prime}\sigma} c_{\mathbf{k}\sigma} \\ &+ \sum_q \omega_q a^{+}_q a_q + J \sum_{q,k} [a^{+}_q c^{+}_{k+q\downarrow} c_{k\uparrow} + h.c.] \end{aligned} \tag{5}$$

where c_k, c^{+}_k are electron field operators and a_q, a^{+}_q are spin-wave operators and J is the effective spin exchange interaction. The spin wave is characterized by $\omega_q = \Delta_g + Aq^2$, where $\Delta_g \approx \mu_B B_{ext} \approx 1K/Tesla$ is the spin-wave gap and $A \approx J/k_F^2$ is the spin stiffness. Later we will drop Δ_g as $\Delta_g < T$. The exchange splitting $B \approx Jk_F^2$ is large, but $B/\epsilon_F << 1$, where we have used the values $B = 700meV$ at 20K [12] and $\epsilon_F = 3.4eV$ [5]. The spin-wave propagator is

$$S^{SW}_{\uparrow\downarrow}(q, \omega_n) = 1/(i\omega_n - a\omega_q) = [S^{SW}_{\downarrow\uparrow}]^* \tag{6}$$

where $\omega_n = 2\pi nT$ is the bosonic Matsubara frequency, $a = 1 - i\gamma/2$ and γ is a phenomenological damping constant. The spin-wave mediated effective interaction is given by $V_{sw}(q, \omega_n) = nJ^2[S^{SW}_{\uparrow\downarrow}(q, \omega_n) + S^{SW}_{\downarrow\uparrow}(q, \omega_n)]$ which is attractive. Here n is the density of conduction electrons.

For the quantum corrections to the conductivity, the dominant contributions from spin-wave interactions come from the diagrams with the most number of diffusons, analogous to those relevant for the Coulomb interactions. We therefore first evaluate the diffuson propagator $\Gamma^{\uparrow\downarrow}(q, \omega)$ in the presence of a large exchange splitting, and obtain

$$\Gamma^{\uparrow\downarrow}(q, \omega_n) = 1/[(2\pi N_0 \tau\hat{\tau})(\omega_n - iB + \hat{D}q^2)] \tag{7}$$

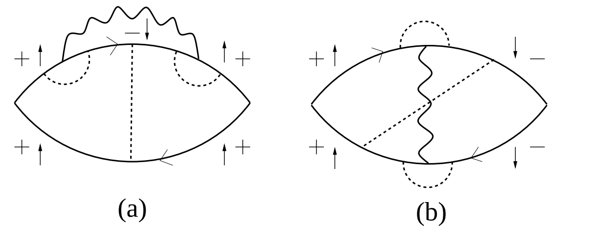

FIG. 3: Spin-wave contributions to the longitudinal conductivity. Solid lines are impurity averaged Green's functions, broken lines are diffusons, wavy line represents the effective spin-wave mediated interactions. There are two diagrams of type (a) and four diagrams of type (b)

where $N_0 = m/2\pi$ is the density of states at the Fermi surface with m being the electron mass, τ is the scattering time, and we have defined

$$\hat{D} \equiv D\,(\hat{\tau}/\tau)^2\,; \quad 1/\hat{\tau} \equiv 1/\tau + \omega_n - iB \tag{8}$$

where $D = (1/2)v_F^2\tau$ is the diffusion coefficient. The corresponding $\Gamma^{\downarrow\uparrow}(q, \omega_n)$ is obtained by replacing B by $-B$ everywhere.

From the diagrams shown in Figure 3, the spin wave contribution to the longitudinal conductivity is given, e.g. for diagram 3a, by

$$\begin{aligned} \delta\sigma^{SW}_{xx\uparrow\downarrow} &= T \sum_{\omega_n} \int_0^{q_c} \frac{dq^2}{(2\pi)^2} q^2 v_F^2 (2\pi N_0 \tau\hat{\tau}^2)^2 (2\pi N_0 \tau)^2 \\ &\times \left[\Gamma^{\uparrow\downarrow}(q, \omega_n)\right]^3 S^{SW}_{\uparrow\downarrow}(q, \omega_n) \end{aligned} \tag{9}$$

and similarly for the $\downarrow\uparrow$ case. Here the upper cut-off in the q-integral is given by $q_c = |1/v_F\hat{\tau}|$. The total spin-wave contribution can then be written as

$$\frac{\delta\sigma^{SW}_{xx}}{L_{00}} \approx \frac{4N_0 J}{1 + \gamma^2/4} \frac{nJ}{B} \frac{\epsilon_F}{B} (\epsilon_F\tau) \frac{T}{Ak_F^2} \tag{10}$$

where we have assumed that $B\tau >> 1$.

Using $n = k_F^2/4\pi$, we estimate $\delta\sigma_{xx}/L_{00} \approx (\frac{Jk_F^2}{2\pi B})^2 (\epsilon_F\tau) \frac{T}{Ak_F^2}$ for small damping. With an estimate of $\epsilon_F\tau \sim 10$ and $(Jk_F^2)/(2\pi B) \sim 1$, the observed magnitude of the constant $P_3 \sim 1$ is consistent if $T_0/Ak_F^2 \sim 1/10$, which is quite reasonable. On the other hand, the disorder dependence of the linear T contribution is given by $P_3 \sim \epsilon_F\tau$, which decreases with increasing disorder. However, as observed experimentally, P_3 appears to decrease only weakly with disorder up to a sheet resistance $R_0 = 2613\Omega$. The variation in sheet resistance in the regime considered here may be mainly due to the change in film thickness, while the disorder strength varies only weakly. We note that while the linear T behavior is observed to be quite robust for weak disorder, it cannot explain the data for $R_0 > 4k\Omega$ (not shown in this paper). We expect that for higher sheet resistances the system will undergo an Anderson localization transition from the

pseudo-metallic phase (localization length longer than the sample dimensions) to a truely localized phase. The study of that regime is the subject of a forthcoming publication.

In conclusion, we have studied charge transport in ultrathin films of Gd grown using *in-situ* techniques which exclude in particular unwanted oxidation or contamination. In addition to the logarithmic temperature dependence expected from the weak localization effects in the longitudinal conductivity as previously seen in Fe films, we observe an additional contribution to the conductivity that has an approximately linear T-dependence for sheet resistance $R_0 < 4k\Omega$ and temperatures $5K < T < 30K$. We interpret this novel feature in terms of contributions from scattering off spin-waves which are known to be important in ferromagnetic films. We find from our calculations that the interaction of the electrons with spin waves of the ordered ferromagnet gives rise to a much larger contribution than the usual one generated by the Coulomb interaction. The temperature dependence is governed by the singular (in the limit $\omega, q \to 0$) spin wave propagator. The dressing by diffuson lines changes the prefactor, but leaves the temperature dependence unchanged. This is, to our knowledge, the first time that this type of quantum correction has been seen and explained.

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